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Probabilistic soft sets and dual probabilistic soft sets in decision making with positive and negative parameters

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Abstract. In this paper, we motivate and introduce probabilistic soft sets and dual probabilistic soft sets for handling decision making problem in the presence of positive and negative parameters. We propose several types of algorithms related to this problem. Our procedures are flexible and adaptable. An example on real data is also given.

1. Introduction

Decision making with positive and negative attributes has been investigated carefully in various fields. For example, in consumer relationship management, a company collects information about consumers complaints to get to know the market sensing [1]. Another example, one needs to combine the opposite criteria such as benefits versus costs, and opportunities versus risks using analytic hierarchy process approach to solve it [2].

One of the theories that can handle decision making problem is soft set theory. The main advantage of soft set is its high flexibility in describing problem. In soft set, we may use various forms of parameterization to describe an object [3]. Many scholars proposed various extended soft sets to deal with decision making problems under uncertainty [4-14]. Nevertheless, soft set theory has not been applied in decision making with positive and negative parameters until what Alcantud & Mathew [6] did. They proposed separable fuzzy soft sets. Fatimah et al. [7] explicated that probabilistic soft sets and dual probabilistic soft sets could be considered as fuzzy soft sets with an additional structure. Hence, we propose several algorithms of the probabilistic soft sets and dual probabilistic soft sets for handling this issue.

This paper is organized as follows. Section 2 recalls the basic definitions of soft set, probabilistic soft set, and dual probabilistic soft set. In Section 3, we propose decision making algorithms for positive and negative parameters using probabilistic soft sets and dual probabilistic soft sets. An application oriented real data is given in Section 4. We conclude in Section 5.

2. Soft set, probabilistic soft set, and dual probabilistic soft set

In this section, we recall the definitions of soft set, probabilistic soft set, and dual probabilistic soft set. Let U as a set of objects, E be a set of parameters where U, E are nonempty finite sets, and $A \subseteq E$.

Definition 1 [3] A soft set (F, A) over U is defined as a mapping from set A to the power set of U, i.e. $F: A \rightarrow 2^U$.

In other words, a soft set is considered as a parameterized family of subsets of U. Considering an example [3], U is a set of houses and E be a set of parameters as follows: expensive, beautiful, wooden, cheap, in the green surroundings, modern, in good repair, and in bad repair. Soft set (F, E) means that the characteristic of houses U is evaluated based on parameters E. In this example, we can see that some parameters may be regarded as negative i.e. 'expensive', and 'in bad repair'.

Zhu [14] introduced an extended model of soft set theory combining probabilistic and soft set. It was called a probabilistic soft set. Its definition as follows.

Definition 2 [14] A probabilistic soft set (F, A) over U is defined as a mapping from set A to the power set of probability distributions $D(U)$, i.e. $F: A \rightarrow D(U)$.

Equivalenty [7], a probabilistic soft set is defined by $F(e_j) \in D(U)$, $\forall e_j \in A$. Therefore for $i = 1,2, \dots, m$ and $j = 1,2, \dots, n$, it can be stated as $F(e_j) = P(a_{ij})/(u_i)$ where $\sum_{i=1}^{m} P(a_{ij}) = 1$, $0 \leq P(a_{ij}) \leq 1$.

Definition 3 [7] A dual probabilistic soft set (F, U) over A is defined as a mapping from set U to the power set of probability distributions $D(A)$, i.e. $F: U \rightarrow D(A)$.

Thus, a dual probabilistic soft set can be denoted by $F(u_i) \in D(A)$, $\forall u_i \in U$. It implies that $F(u_i) = P(a_{ij})/(e_j)$ where $\sum_{j=1}^{n} P(a_{ij}) = 1$, $0 \leq P(a_{ij}) \leq 1$.

Soft set, probabilistic soft sets, and dual probabilistic soft sets could be represented in tabular forms. Rows indicate objects U, and columns indicate parameters A. In a soft set, all cells are either 0 or 1. Then, in probabilistic soft sets and dual probabilistic soft sets, all cells are in interval $[0,1]$ with requirements as in the explanation of Definitions 2 & 3 mentioned above.

3. Generalization of probabilistic soft sets and dual probabilistic soft sets algorithms

Fatimah et al [7] discussed about the concepts of probabilistic soft sets and dual probabilistic soft sets. They proposed decision making algorithms which were appropriate for positive parameters. In relation to this assumption, decision makers may meet with both positive and negative parameters or just consider negative ones. In order to accommodate all needs of the decision makers, we introduce generalization of probabilistic soft sets and dual probabilistic soft sets algorithms which are extension forms of the algorithms in Fatimah et al [7]. These mechanisms are applicable for both positive and negative parameters.

All our algorithms use the same first step (input) as follows.

Step 1. Input a set of objects $U = \{u_i, i = 1,2, \dots, m\}$, and a set of parameters $E = \{e_j, j = 1,2, \dots, n\}$, $A \subseteq E$.

i. For the probabilistic soft sets, input a table of (F, A) and let $P(a_{ij}) = F(e_j)(u_i)$ be the entries of its tabular representation whereby $\sum_{i=1}^{m} P(a_{ij}) = 1$ for $j = 1,2, \dots, n$.

ii. For the dual probabilistic soft sets, input a table of (F, U) and let $P(a_{ij}) = F(u_i)(e_j)$ be the entries of its tabular representation whereby $\sum_{j=1}^{n} P(a_{ij}) = 1$ for $i = 1,2, \dots, m$.

Algorithm 1 Generalization of Probabilistic Soft Sets-Choice Values (GPSS-CV)

Step 2. For all $i = 1,2, \dots, m$, find choice values (c_i) by using an operation table as follows:

i. If A means a set of positive parameters then $c_i = \sum_{j=1}^{n} P(a_{ij})$.

ii. If A means a set of negative parameters then $c_i = \sum_{j=1}^{n} -P(a_{ij})$.

iii. If A has both positive and negative parameters then $c_i = c_{pos} + c_{neg}$ where $c_{pos} = \sum_{r} P(a_{ij})$ and $c_{neg} = \sum_{s} -P(a_{ij})$ for all r positive parameters and s negative parameters.

Step 3. Find the decision:

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i. For a single decision, find k for which $c_k = max_{i=1,\dots,m} c_i$. Then u_k is the optimal choice object. If c_i attains its maximum value at more than one index k, then any one of them could be chosen by decision maker.
ii. For multiple decisions, rank decisions u_i from the highest to lowest c_i.

Algorithm 2 Generalization of Probabilistic Soft Sets-Minimax (GPSS-M)
Step 2. Find M_j as follows:
i. If A means a set of positive parameters then M_j is the maximum for each column e_j.
ii. If A means a set of negative parameters then M_j is the minimum for each column e_j.
iii. If A has both positive and negative parameters then M_j is the maximum for each column e_j which means positive and M_j is the minimum for otherwise.
Step 3. Make a new table where every cell is obtained from the previous table (Step 1) by subtracting the corresponding cell in that table from the M_j at its column.
Step 4. Find the maximum of each row u_i. It is denoted by $Minimax_i$.
Step 5. Find the decision:
i. For a single decision, find k for which $Minimax_k = min_{i=1,\dots,m} Minimax_i$. Then u_k is the optimal choice object. If we obtain more than one index k then any one of u_k could be chosen by decision maker.
ii. For multiple decisions, rank decisions u_i from the lowest to highest $Minimax_i$.

Algorithm 3 Generalization of Probabilistic Soft Sets-Opportunity Cost (GPSS-OC)
Step 2. Find M_j as follows:
i. If A means a set of positive parameters then M_j is the maximum for each column e_j.
ii. If A means a set of negative parameters then M_j is the minimum for each column e_j.
iii. If A has both the positive and negative parameters then M_j is the maximum for each column e_j which means positive and M_j is the minimum for otherwise.
Step 3. Make a new table where every cell is obtained from the previous table (Step 1) by subtracting the corresponding cell in that table from the M_j at its column.
Step 4. Find the opportunity cost (OC_i) values as a sum of each row u_i for $i = 1,2,\dots,m$.
Step 5. Find the decision:
i. For a single decision, find k for which $OC_k = min_{i=1,\dots,m} OC_i$. Then u_k is the optimal choice object. If we obtain more than one index k then any one of u_k could be chosen by decision maker.
ii. For multiple decisions, rank decisions u_i from the lowest to highest OC_i.

Algorithm 4 Generalization of Probabilistic Soft Sets-Weighted Choice Values (GPSS-WCV)
Step 2. Make a table according to the weighted parameters $W = \{w_j, j = 1,2,\dots,n\}$ i.e., $P_{w_j}(a_{ij}) = P(a_{ij}) \times w_j, \forall j$.
Step 3. For all $i = 1,2,\dots,m$, find weighted choice values (wc_i) by using an operation table as follows:
i. If A means a set of positive parameters then $wc_i = \sum_{j=1}^{n} P_{w_j}(a_{ij})$.
ii. If A means a set of negative parameters then $wc_i = \sum_{j=1}^{n} -P_{w_j}(a_{ij})$.
iii. If A has both positive and negative parameters then $wc_i = wc_{pos} + wc_{neg}$ where $wc_{pos} = \sum_r P_{w_j}(a_{ij})$ and $wc_{neg} = \sum_s -P_{w_j}(a_{ij})$ for all r positive parameters and s negative parameters.
Step 2. Find the decision:

i. For a single decision, find k for which $wc_k = max_{i=1,\ldots,m} wc_i$. Then u_k is the optimal choice object. If we obtain more than one index k then any one of u_k could be chosen by decision maker.
ii. For multiple decisions, rank decisions u_i from the highest to lowest wc_i.

Algorithm 5 Generalization of Probabilistic Soft Sets-Weighted Minimax (GPSS-WM)
Step 2. Make a table according to the weighted parameters $W = \{w_j, j = 1,2,\ldots,n\}$ i.e., $P_{w_j}(a_{ij}) = P(a_{ij}) \times w_j, \forall j$.
Step 3. Find wM_j as follows:
i. If A means a set of positive parameters then wM_j is the maximum for each column e_j.
ii. If A means a set of negative parameters then wM_j is the minimum for each column e_j.
iii. If A has both the positive and negative parameters then wM_j is the maximum for each column e_j which means positive and wM_j is the minimum for otherwise.
Step 4. Make a new table where every cell is obtained from the previous table (Step 1) by subtracting the corresponding cell in that table from the wM_j at its column.
Step 5. Find the maximum of each row u_i. It is denoted by $wMinimax_i$.
Step 6. Find the decision:
i. For a single decision, find k for which $wMinimax_k = min_{i=1,\ldots,m} wMinimax_i$. Then u_k is the optimal choice object. If we obtain more than one index k then any one of u_k could be chosen by decision maker.
ii. For multiple decisions, rank decisions u_i from the lowest to highest $wMinimax_i$.

Algorithm 6 Generalization of Probabilistic Soft Sets-Weighted Opportunity Cost (GPSS-WOC)
Step 2. Make a table according to the weighted parameters $W = \{w_j, j = 1,2,\ldots,n\}$ i.e., $P_{w_j}(a_{ij}) = P(a_{ij}) \times w_j, \forall j$.
Step 3. Find wM_j as follows:
i. If A means a set of positive parameters then wM_j is the maximum for each column e_j.
ii. If A means a set of negative parameters then wM_j is the minimum for each column e_j.
iii. If A has both the positive and negative parameters then wM_j is the maximum for each column e_j which means positive and wM_j is the minimum for otherwise.
Step 4. Make a new table where every cell is obtained from the previous table (Step 1) by subtracting the corresponding cell in that table from the wM_j at its column.
Step 5. Find the weighted opportunity cost (wOC_i) values as a sum of each row u_i for $i = 1,2,\ldots,m$.
Step 6. Find the decision:
i. For a single decision, find k for which $wOC_k = min_{i=1,\ldots,m} wOC_i$. Then u_k is the optimal choice object. If we obtain more than one index k then any one of u_k could be chosen by decision maker.
ii. For multiple decisions, rank decisions u_i from the lowest to highest wOC_i.

Algorithm 7 Generalization of Probabilistic Soft Sets & Dual Probabilistic Soft Sets-Positive Matrices
Step 2. For all $i = 1,2,\ldots,m$, and p_{ij} denotes every cell (i,j) then use an operation table as follows:
i. If e_j is a negative parameter then $p_{ij} = -P(a_{ij})$.
ii. If e_j is a positive parameter then $p_{ij} = P(a_{ij})$.
Step 3. Construct a matrix $C = (c_{ij})_{m \times m}$ where:
i. If $i \neq j$, c_{ij} is the number of parameters for which the value of u_i is strictly greater than the value of u_j. Thus, c_{ij} is the number of parameters j for which $p_{ij} - p_{mj} > 0$, or the number of positive values in the finite sequence $p_{i1} - p_{m1}, p_{i2} - p_{m2}, \ldots, p_{in} - p_{mn}$.

ii. If $i = j$, $c_{ij} = n(m-1) - t_j$ where $t_j = \sum\{c_{qj}: q \neq j, q = 1, \dots, m\}$ is the sum of the nondiagonal elements in column j of C. This means that we define c_{ii} as the number such that column i in C sums up to $n(m-1)$.

Step 4. Compute one eigenvector $H = (H_1, \dots., H_k)$ associated with the dominant eigenvalue of the matrix which is $n(m-1)$.

Step 5. Find the decision:

i. For a single decision, find k for which $H_k = max_{i=1,\dots,m} H_i$. Then u_k is the optimal choice object. If H_i attains its maximum value at more than one index k, then any one of them could be chosen by decision maker.

ii. For multiple decisions, rank decisions u_i from the highest to lowest H_i.

4. An application oriented real data sets

In the following example, we use a real case study to apply one of our algorithms. The real data sets are from Statistical Yearbook of Indonesia 2016, BPS-Statistics Indonesia [15].

Table 1 Distribution of household's population

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	Total
u_1	0.064	0.056	0.345	0.313	0.092	0.064	0.023	0.039	0.004	0.001	1
u_2	0.143	0.180	0.329	0.135	0.047	0.079	0.032	0.030	0.023	0.001	1
u_3	0.117	0.050	0.345	0.206	0.079	0.100	0.068	0.017	0.017	0.002	1
u_4	0.005	0.104	0.460	0.164	0.070	0.010	0.004	0.009	0.174	0.000	1
u_5	0.106	0.026	0.226	0.286	0.178	0.011	0.009	0.053	0.104	0.001	1
u_6	0.172	0.058	0.182	0.351	0.105	0.020	0.013	0.045	0.053	0.001	1
u_7	0.090	0.042	0.136	0.241	0.360	0.059	0.051	0.016	0.002	0.002	1
u_8	0.021	0.067	0.187	0.421	0.228	0.036	0.027	0.007	0.006	0.000	1
u_9	0.016	0.085	0.520	0.275	0.086	0.002	0.009	0.003	0.003	0.000	1
u_{10}	0.116	0.010	0.661	0.121	0.050	0.023	0.015	0.002	0.002	0.000	1
u_{11}	0.144	0.145	0.706	0.004	0.000	0.000	0.000	0.000	0.000	0.001	1
u_{12}	0.070	0.211	0.352	0.193	0.038	0.092	0.040	0.003	0.001	0.001	1
u_{13}	0.148	0.176	0.169	0.298	0.038	0.129	0.031	0.004	0.006	0.001	1
u_{14}	0.115	0.086	0.223	0.470	0.026	0.029	0.007	0.000	0.044	0.000	1
u_{15}	0.095	0.237	0.237	0.259	0.024	0.121	0.020	0.004	0.003	0.000	1
u_{16}	0.053	0.258	0.462	0.108	0.048	0.029	0.033	0.004	0.003	0.000	1
u_{17}	0.256	0.070	0.390	0.052	0.010	0.162	0.021	0.010	0.031	0.000	1
u_{18}	0.143	0.123	0.157	0.389	0.036	0.128	0.015	0.006	0.002	0.001	1
u_{19}	0.139	0.036	0.051	0.184	0.062	0.329	0.116	0.038	0.044	0.002	1
u_{20}	0.031	0.032	0.164	0.057	0.047	0.089	0.047	0.119	0.413	0.002	1
u_{21}	0.083	0.137	0.335	0.077	0.071	0.012	0.016	0.189	0.082	0.000	1
u_{22}	0.307	0.108	0.225	0.077	0.134	0.010	0.002	0.117	0.020	0.000	1
u_{23}	0.227	0.027	0.596	0.031	0.032	0.014	0.016	0.037	0.022	0.000	1
u_{24}	0.113	0.025	0.513	0.024	0.013	0.018	0.009	0.061	0.219	0.004	1
u_{25}	0.097	0.074	0.370	0.204	0.051	0.175	0.016	0.003	0.011	0.000	1
u_{26}	0.093	0.157	0.202	0.133	0.047	0.247	0.040	0.080	0.003	0.000	1
u_{27}	0.153	0.177	0.253	0.194	0.059	0.098	0.042	0.012	0.013	0.000	1
u_{28}	0.136	0.097	0.205	0.288	0.050	0.152	0.031	0.013	0.028	0.001	1
u_{29}	0.159	0.112	0.262	0.318	0.048	0.062	0.022	0.016	0.000	0.002	1
u_{30}	0.091	0.101	0.150	0.206	0.071	0.150	0.115	0.111	0.005	0.001	1
u_{31}	0.123	0.083	0.136	0.287	0.047	0.260	0.034	0.009	0.021	0.001	1
u_{32}	0.176	0.029	0.139	0.337	0.101	0.148	0.016	0.036	0.019	0.000	1
u_{33}	0.064	0.031	0.365	0.144	0.045	0.081	0.070	0.075	0.125	0.000	1
u_{34}	0.058	0.016	0.203	0.051	0.033	0.127	0.272	0.087	0.153	0.001	1

Let a set of provinces in Indonesia, $\boldsymbol{U}$ = {Aceh ($\boldsymbol{u_1}$), Sumatera Utara ($\boldsymbol{u_2}$), Sumatera Barat ($\boldsymbol{u_3}$), Riau ($\boldsymbol{u_4}$), Jambi ($\boldsymbol{u_5}$), Sumatera Selatan ($\boldsymbol{u_6}$), Bengkulu ($\boldsymbol{u_7}$), Lampung ($\boldsymbol{u_8}$), Kepulauan Bangka Belitung ($\boldsymbol{u_9}$), Kepulauan Riau ($\boldsymbol{u_{10}}$), DKI Jakarta ($\boldsymbol{u_{11}}$), Jawa Barat ($\boldsymbol{u_{12}}$), Jawa Tengah ($\boldsymbol{u_{13}}$), DI Yogyakarta ($\boldsymbol{u_{14}}$), Jawa Timur ($\boldsymbol{u_{15}}$), Banten ($\boldsymbol{u_{16}}$), Bali ($\boldsymbol{u_{17}}$), Nusa Tenggara Barat ($\boldsymbol{u_{18}}$), Nusa Tenggara Timur

($\boldsymbol{u_{19}}$), Kalimantan Barat ($\boldsymbol{u_{20}}$), Kalimantan Tengah ($\boldsymbol{u_{21}}$), Kalimantan Selatan ($\boldsymbol{u_{22}}$), Kalimantan Timur ($\boldsymbol{u_{23}}$), Kalimantan Utara ($\boldsymbol{u_{24}}$), Sulawesi Utara ($\boldsymbol{u_{25}}$), Sulawesi Tengah ($\boldsymbol{u_{26}}$), Sulawesi Selatan ($\boldsymbol{u_{27}}$), Sulawesi Tenggara ($\boldsymbol{u_{28}}$), Gorontalo ($\boldsymbol{u_{29}}$), Sulawesi Barat ($\boldsymbol{u_{30}}$), Maluku ($\boldsymbol{u_{31}}$), Maluku Utara ($\boldsymbol{u_{32}}$), Papua Barat ($\boldsymbol{u_{33}}$), Papua ($\boldsymbol{u_{34}}$)}. Consider a set of drinking water sources $\boldsymbol{E}$ = {piped water ($\boldsymbol{e_1}$), pumped water ($\boldsymbol{e_2}$), bottled water ($\boldsymbol{e_3}$), protected well ($\boldsymbol{e_4}$), unprotected well ($\boldsymbol{e_5}$), protected spring ($\boldsymbol{e_6}$), unprotected spring ($\boldsymbol{e_7}$), surface water ($\boldsymbol{e_8}$), rainwater collection ($\boldsymbol{e_9}$), other ($\boldsymbol{e_{10}}$)}. The negative parameters are $\boldsymbol{e_5}, \boldsymbol{e_7}, \boldsymbol{e_8}$, and $\boldsymbol{e_{10}}$. Distribution of households population according to province and source of drinking water in 2015 are described in Table 1.

Therefore, Table 1 is a dual probabilistics soft sets (cf., Definition 3). We run the code of Algorithm 7 using R version 3.3.1, PC Intel(R)core(TM)-i3 with 4GB RAM, and Windows 7 as operating system. The eigen dominan is 330 and the optimal province is DI Yogyakarta. The top 6 provinces are $\boldsymbol{u_{14}} > \boldsymbol{u_{17}} > \boldsymbol{u_{15}} > \boldsymbol{u_{13}} > \boldsymbol{u_{25}} > \boldsymbol{u_{11}}$.

5. Conclusion

In this paper, we have acquainted decision making algorithms of probabilistic soft sets and dual probabilistic soft sets for positive and negative parameters. We believe that our algorithms can be applied either in illustrative examples or in real data sets. We also believe that these procedures deserve farther studies such as comparison with separable fuzzy soft sets.

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